Further Pure Core – Volumes Of Revolution

1. The curve $y = 2\sqrt{x}$ is rotated about the x–axis through 360° between $x = 0$ and $x = 3$. Find the volume generated. $\boxed{18\pi}$

2. The area under the curve $y = (2x - 1)^3$ between $x = \frac{1}{2}$ and $x = 2$ is rotated through 360° about the x–axis. Find the volume of revolution. $\boxed{\frac{2187\pi}{14}}$

3. Sketch the graph $y = e^x + 1$. The area under the curve between $x = 0$ and $x = \ln 2$ is rotated about the x–axis through 360°. Calculate the volume generated. □

4. Sketch the graph of $y = x^2 + 1$. Show that the area between the curve and the y–axis between $y = 1$ and $y = 3$ is given by
$$\int_1^3 \sqrt{y-1}\,dy$$
and calculate it. $\boxed{\frac{4\sqrt{2}}{3}}$

   If this area is rotated about the y–axis through 360°, calculate the volume generated. $\boxed{2\pi}$

5. The curve $y = \ln x$ is rotated about the y–axis through 360° between $y = 0$ and $y = 2$. Find the volume of revolution. $\boxed{\frac{\pi(e^4-1)}{2}}$

6. Sketch the graphs of $y = 3x$ and $y = x^2$. Find the area between the line $y = 3x$ and curve $y = x^2$. $\boxed{\frac{9}{2}}$

   Find the volume of revolution if the area is rotated through 360° about the

   (a) x–axis, $\boxed{\frac{162\pi}{5}}$

   (b) y–axis. $\boxed{\frac{27\pi}{2}}$

7. Sketch on the same diagram $y = 3 - e^x$ and $y = 2e^{-x}$. If the finite area enclosed between them is rotated through 360° about the x–axis, find the volume generated. $\boxed{3\pi(3\ln 2 - 2)}$

8. A cylindrical hole is drilled through a sphere, with the axis of the cylinder passing through the centre of the sphere. The length of the hole is $2a$. Use calculus to prove that the volume of the part of the sphere that remains does not depend on the radius of the sphere. □